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The Complexity of One of Subsystem and The Set-valued System Induced by It on Symbolic Spaces

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Abstract

Let $(\sum;\rho)$ be a one-sided symbolic space, and let σ be the shift on Σ. In this paper, we prove that there exist a subset $J\subset\sum$ such that $(\kappa(J);\overline{\sigma})$ is M-systems, where $(\kappa(J);\overline{\sigma})$ be the set-valued dynamical system induced by σ.

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1. Introduction

It is well known that the central problem of dynamical system is to investigate asymptotic behavior and topological structure of the orbits of points. Motional properties of points as a whole in the space can reflect better the dynamical character of the whole system. So it should be paid attention to the subjects of topological mixing, topological transitivity and ergodicity in researching dynamical system. However, when handling problems such as numerical simulation, attractors, migration and species breed, it is not enough to know how some single individualities (i.e. some points in the space) change, it needs to know how some groups (i.e. some subsets in the space) change. Therefore, the study of set-valued system induced by single-valued system has attracted more and more scholar's attentions. Dynamical properties of the kind of set-valued discrete system completely depend on the single-valued system inducing it. It is

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inevitable that dynamical properties of the set-valued system reflect those of the single-valued system , and they link closely. So set-valued discrete system offers a new way to study dynamical properties of the single-system.

First of all, R. F. Heriberto [1] discussed and defined the relationships between continuous map on the metric space and transitivity of the corresponding expansion map on set-valued discrete system induced by the continuous map. Then he proposed "whether does the chaos of f imply the chaoticity of $\overline{f}$ ". Drawing inspiration from this, A. Fsdeli [2] discussed the relationship between continuous map on the constant subsystem of set-valued discrete system and transitivity of the continuous map on basic system and made his findings that, under certain conditions, Devaney chaos of the expanding map on constant subsystem of set-valued discrete system is equivalent to that of the continuous map on basic system. Meanwhile, he proposed "when dose the chaos of f imply the chaoticity of $\overline{f}$ ". Liao, Zhang and the author of this paper discussed and answered the two questions in [3].

Besides, symbolic dynamical system as a research direction has been a useful tool for giving the proof of existence and constructing the counterexamples to investigate the complexity of system. The main purpose of this paper is to construct a M-system on symbolic space and explicate that the set-valued system induced by it is also a M-system.

The main results are stated as follows:

Let $(\sum, \rho)$ be a one-sided symbolic space with two symbolic symbols and σ be the shift on it. There exists a subset $J \subset \sum$, with the following properties:

- (P1) (J ; σ) is M-system;
- (P2) (J ; σ) is topologically ergodic;
- (P3) (J ; σ) is topologically double ergodic;
- (p4) ($\kappa(J)$; $\overline{\sigma}$) is topologically weakly mixing;
- (P5) ($\kappa(J)$; $\overline{\sigma}$) is almost periodically dense;
- (P6) ($\kappa(J)$; $\overline{\sigma}$) is M-system.

2 . Basic definitions and preparations

Definition 2.1. Let U, V be any non-empty open set of X , (*i*) f is said to be topologically ergodic if

$$\overline{\lim_{n\to\infty}}\frac{1}{n}[\#(\mathrm{K}(U,V)\bigcap\{0,1,\cdots,n-1\})]>0$$

i.e. $\mathrm{K}(U,V)$ is a positive upper density subset of the set of positive integers. (*ii*) f is said to be topologically double ergodic if $f \times f$ is topologically ergodic.

*Definition 2.2.*Let $S=\{0,1\}$, $\sum=\{x=x_1x_2\cdots \mid x_i\in S, i=1,2,\cdots\}$. Define $\rho:\sum\times\sum\to R$ as follows: For any $x, y\in\sum$, if $x=x_1x_2\cdots$ and $y=y_1y_2\cdots$ then

$$\rho(x,y)=\begin{cases}0 & if \quad x=y\\ \dfrac{1}{k} & if \ x\neq y, \ and \ k=\min\{n\mid x_n\neq y_n\}.\end{cases}$$

It is not difficult to check that ρ is a metric on $\sum$. The space $(\sum,\rho)$ is compact and called the one-sided symbolic space (with two symbols).

Define $\sigma:\sum\to\sum$ by $\sigma=(x_1x_2\cdots)=x_2x_3\cdots$ for any $x=x_1x_2\cdots\in\sum.\sigma$ is continuous and called the shift on $\sum$. The $(\sum,\rho)$ is a compact system. If $Y\subset\sum$ is closed and $\sigma(Y)=Y$, then $\sigma|_Y:Y\to Y$ is called a sub-shift of σ.

Definition 2.3. Call A a tuple (over S={0,1}), if it is a finite arrangement of elements in S. If $A=a_1a_2\cdots a_n$, where $a_i\in S, 1\le i\le n$, then the length of A is said to be n, denoted by $|A|=n$. Let $B=b_1b_2\cdots b_m$ be another tuple.

We say B occurs in A, if there exists $i\ge 0$ such that $b_j=a_{i+j}$ (1), for each $j=1,2,\cdots,m.$ Denote $B\prec A.$ The number of i satisfying (1) is called the occurrence number of B in $A,$ and is denote by $L_B(A).$

Foy any tuple $B=b_1b_2\cdots b_n$, denote $[B]=\{x=x_1x_2\cdots\in\sum\mid x_i=b_i, 1\le i\le n\}$ which will be called a cylinder generated by B.

Lemma 2.1. Let (X,f) be a compact system. $f:X\to X$ is minimal if and only if for any $x\in X,\{f^n(x)\mid n\in N\}=X.$

Proof: For a proof, see Ref.[4].

Lemma 2.2. Let (X,f) be a compact system. The following are equivalent:

$(i)x\in A(f),(ii)x\in\omega(X,f)$ is a minimal set.

Proof: For a proof, see Ref.[5][6].

Lemma 2.3. The following are equivalent:

$(i)f$ is weakly mixing; (ii) for any $m\ge 2, f_m$ is transitive; (iii) for any non-empty open sets U and V, there is an $n>0$ such that $f^n(U)\bigcap V\neq\Phi$ and $f^n(V)\bigcap V\neq\Phi$.

Proof: For a proof, see Ref.[3].

Lemma 2.4. Let (X,f) be a dynamical system, then $A(f)\neq\Phi$.

Lemma 2.5 Let $(X_1\times X_2\times\cdots\times X_n, f_1\times f_2\times\cdots\times f_n)$ be the product system constituted by $(X_i,f_i), i=1,2,\cdots,n,$ if $(x_1,x_2,\cdots,x_n)$ is a almost periodic point of $f_1\times f_2\times\cdots\times f_n$, then so is $(f_1^{m_1}(x_1)f_2^{m_2}(x_2),\cdots,f_n^{m_n}(x_n)),$ for any $m_i\in N, i=1,2,\cdots,n.$

Proof. Let $(x_1,x_2,\cdots,x_n)$ be a almost periodic point of $f_1\times f_2\times\cdots\times f_n$, for any $\varepsilon>0$ and $m_i\in N, i=1,2,\cdots,n,$ let $q=\max\{m_1,m_2,\cdots,m_n\}.$ By continuities of f_i, there exists $\delta>0$ such that $d(f_i^{\,j}(x),f_i^{\,j}(y))<\varepsilon$ for all $d(x,y)<\delta, x,y\in X_i, j=1,2,\cdots,q$ and $i=1,2,\cdots,n.$

Since $(x_1,x_2,\cdots,x_n)$ is a almost point, $N((x_1,x_2,\cdots,x_n),B((x_1,x_2,\cdots,x_n),\delta))$ is syndetic, so $N((f_1^{m_1}(x_1), f_2^{m_2}(x_2), \cdots f_n^{m_n}(x_n)), B((f_1^{m_1}(x_1), f_2^{m_2}(x_2), \cdots f_n^{m_n}(x_n)), \varepsilon))$ is syndetic too.

3. Proof of Main Theorem

In this section we shall prove the theorem. For this we first restate the construction of the sub-shift as follows:

Let $A = a_1 a_2 \cdots a_n$ be a tuple over $S = \{0,1\}$.Define the inverse of A to be $\overline{A} = \overline{a}_1 \overline{a}_2 \cdots \overline{a}_n$, where for i=1,2,…,n,

$$\overline{a}_i = \begin{cases} 0 \, if \ a_i = 1 \\ 1 \, if \ a_i = 0 \end{cases}$$

Take an arbitrary tuple A_1. Let A_2 be an arrangement of A_1 and $\overline{A}_1$, say $A_2 = A_1 \overline{A}_1$ (or $\overline{A}_1 A_1$). Define inductively the tuples $A_2, A_3, \cdots$ such that A_n is a arrangement of all the tuples of the finite set

$$\wp_{\text{n-1}} = \{J_1 J_2 \cdots J_{n-1} \mid j_i \in \{A_i, \overline{A}_i\}, 1 \le i \le n-1\}$$

Let $a = A_1 A_2 \cdots$, By the proof of [7] we know that $a \in A(\sigma)$, then $a \in \omega(a,\sigma)$ and $\omega(a,\sigma)$ is a minimal set.

Choose an uncountable subset E in Σ such that for any different points $x = x_1 x_2, \cdots, y = y_1 y_2 \cdots$, both $x_n = y_n$ holds for infinitely many n and $x_m \ne y_m$ holds for infinitely many m. Such a subset exists, cf.,e.g.,[8]. Define $\varphi : E \to \sum$, such that for any $x = x_1 x_2, \cdots \in E$, $\varphi(x) = B_1 B_2 \cdots$, where

$$B_i = \begin{cases} A_i \, if \ x_i = 1 \\ \overline{A}_i \, if \ x_i = 0 \end{cases}$$

There exists $k \ge 0$ such that the first m_i coordinate vector of $\sigma_k(a)$ is $B_1 B_2 \cdots B_i (B_1 B_2 \cdots B_i \models m_i)$. Since for every fixed i, on matter how $B_{\mathrm{j}} (1 \le \mathrm{j} \le \mathrm{i})$ select, $B_1 B_2 \cdots B_i \prec A_{i+1} \prec a$. This shows that for every $x \in E$, $\varphi(x) \in \omega(a,\sigma)$. Let $D = \varphi(E), D \subset \omega(a,\sigma)$. D is an uncountable set since E is an uncountable set and φ is an injection. So $\omega(a,\sigma)$ is an uncountable set. Let $J = \omega(a,\sigma)$, then $J \subset A(\sigma)$.

Now we give the proof of the theorem.

Proof of (P1). By the construction, we can know that (J,σ) is almost periodically dense. Since $\sigma|_J$ is minimal map, it is transitive. Above all, (J,σ) is M-system.

Proof of (P2). Let U, V be non-empty open sets of J and $U,V \subset \overline{A(\sigma)}$. By the proof of (P1), $\sigma|_J$ is topologically transitive. So there is a positive integer n such that $\sigma^{-n}(U)\bigcap V \neq \Phi$. Thus there exists $x\in A(\sigma)\bigcap\sigma^{-n}(U)\bigcap V$, i.e. there exists $x\in V\bigcap A(\sigma)$ and $\sigma^{-n}(x)\in U$, U is an open neighbourhood of $\sigma^n(x)$. Because σ^n is continuous, for the neighbourhood U of $\sigma^n(x)$, there exists $D\subset V$, where D is a neighbourhood of x and $\sigma^n(D)\subset U$. Since $x\in A(\sigma)$, there exists $L>0$ such that

$$\lim_{nL\to\infty}\frac{\#[\{n\mid\sigma^n(U)\bigcap V\neq\Phi\}\bigcap\{0,1,\cdots,nL-1\}]}{nL}\geq\frac{1}{L}$$

So $\sigma|_{\overline{A(\sigma)}}$ is topologically ergodic. By the proof of (P1), (J,σ) is almost periodically dense. Thus $\overline{A(\sigma)}=J$ .Above all, (J,σ) is topologically ergodic.

Proof of (P3). By [9], $\sigma|_J$ is topologically weakly mixing. Since $J=\omega(a,\sigma)$ is a minimal set,by the construction, (J,σ) is a minimal subsystem of σ.

Firstly,we will prove that $(J\times J,\sigma\times\sigma)$ is a minimal subsystem of $\sigma\times\sigma$ if (J,σ) is a minimal subsystem of σ. By lemma 2.1,we only need to show that for $\forall(x,y)\in J\times J$. $\overline{(\sigma\times\sigma)^n((x,y))}=J\times J$

Let $(x,y)\in J\times J$, then $\overline{\{\sigma^n(x)\mid n\in N\}}=J,\overline{\{\sigma^n(y)\mid n\in N\}}=J$. So

$$J\times J=\overline{\{\sigma^n(x)\mid n\in N\}}\times\overline{\{\sigma^n(y)\mid n\in N\}}=\overline{\{\sigma^n(x)\mid n\in N\}\times\sigma^n(y)\mid n\in N\}}.$$

Secondly, we will prove that for $\forall(x,y)\in J\times J$, $(x,y)\in A(\sigma\times\sigma)$ .By lemma 2.2, we only need to show that $(x,y)\in\omega[(x,y),\sigma\times\sigma]$. Since $\sigma|_J$ is topologically weakly mixing, for any neighbourhood $U\times V$, there exists $n>0$ such that $[(\sigma\times\sigma)^n](U\times V)]\bigcap(U\times V)\neq\Phi$ .Further, there exists an increasing sequence $\{p_i\}$ of positive integers such that $\lim_{i\to\infty}(\sigma\times\sigma)^{p_i}((x,y))=(x,y)$

So $(x,y)\in\omega[(x,y),\sigma\times\sigma]$. By lemma 2.2, $(x,y)\in A(\sigma\times\sigma)$.

Finally, we will prove that $\sigma|_J$ is topologically double ergodic. It suffices to prove that $\sigma\times\sigma|_{J\times J}$ is topologically ergodic. This can be known by the above proof and the prove of (P2).

Above all, (J,σ) is topologically double ergodic.

Proof of (P4). By the proof of (P3), $\sigma|_J$ is topologically weakly mixing. Let U, V be non-empty open sets of $\kappa(J)$ .There exist non-empty open sets $U_1, U_2, \cdots U_s, V_1, V_2, \cdots V_t$ of J , such that $B_1 = B(U_1, U_2, \cdots U_s) \subset U, B_2 = B(V_1, V_2, \cdots V_t) \subset V$, Let $m = \max\{s,t\}$,and put

$$\overline{U}_i = \begin{cases} U_i, & 1 \le i \le s \\ U_s, & s \le i \le m \\ V_i, & m+1 \le i \le m+t \\ V_i, & m+t \le i \le 2m \end{cases} \quad \overline{V}_i = \begin{cases} V_i, 1 \le i \le t \; or \, m+1 \le i \le m+t \\ V_t, t \le i \le m \, or \;\; m+t \le i \le 2m \end{cases}$$

Evidently, $\overline{U}_i, \overline{V}_i$ are both non-empty open sets of J . Since $\sigma|_J$ is topologically weakly mixing, by lemma 2.3, σ_{2m} is topologically transitive. So for some $n > 0$, $\sigma^n(\overline{U}_i) \bigcap \overline{V}_i \ne \Phi$ for $1 \le i \le 2m$. Thus for each we can choose $i = 1,2,\cdots,2m$, we can choose $x_i \in \overline{U}_i, y_i \in \overline{V}_i$ such that $\sigma^n(x_i) = y_i$ .Put $E_1 = \{x_1, x_2, \cdots x_m\}, F_1 = \{y_1, y_2, \cdots . y_m\}, E_2 = \{x_{m+1}, x_{m+2}, \cdots x_{2m}\}$, $F_2 = \{y_{m+1}, y_{m+2}, \cdots . y_{2m}\}$. We have $\overline{\sigma}^n(E_i) = F_i, i = 1,2.$ Since we easily see that $E_1 \subset B_1$, and $E_2, F_1, F_2 \subset B_2$, it follows that $\overline{\sigma}^n(U) \bigcap V \supset \overline{\sigma}^n(B_1) \bigcap B_2 \ne \Phi$ and $\overline{\sigma}^n(U) \bigcap V \supset \overline{\sigma}^n(B_2)$ $\bigcap B_2 \ne \Phi$. By lemma 2.3, $\overline{\sigma}$ is topologically weakly mixing.

Above all, $(\kappa(J), \overline{\sigma})$ is topologically weakly mixing.

Proof of (P5). By the proof of (P1), (J, σ) is almost periodically dense. Let $A = \{\mathrm{x}_1, \mathrm{x}_2, \cdots\}$ be a countable dense subset of $A(\sigma)$, so $\overline{A} = \overline{A(\sigma)} = J$,i.e., A is a dense subset of J too. Then for any $K \in \kappa(J)$ and $\varepsilon > 0$, there exists a finite subset $B = \{x_1', x_2', \cdots\} \subset \mathrm{A}$ such that $H(B,K) < \dfrac{\varepsilon}{2}$.

Let $A_i = O(x_i', \sigma), i = 1,2,\cdots,n$, then $A_1, A_2, \cdots A_n$ are minimal. By lemma 2.4, there exists a almost periodic point $(y_1, y_2, \cdots . y_n)$ in the product system $(A_1 \times A_2 \times \cdots \times A_n, \sigma \times \sigma \times \cdots \times \sigma)$. Since A_i is minimal, y_i is transitive in (A_i, σ), for any $i \in \{1,2,\cdots,n\}$. So there exist $m_1, m_2, \cdots . m_n$ such that

$$d(\sigma^{m_i}(y_i), x_i') < \frac{\varepsilon}{2} \text{ for all } i = 1,2,\cdots,n.$$

Therefore, $$H(\sigma^{m_1}(y_1), \sigma^{m_2}(y_2), \cdots, \sigma^{m_n}(y_n)), (x_1', x_2', \cdots x_n')) < \frac{\varepsilon}{2}.$$

Hence $H(\sigma^{m_1}(y_1),\sigma^{m_2}(y_2),\cdots,\sigma^{m_n}(y_n)),K)<\varepsilon$.

By lemma 2.5, $(\sigma^{m_1}(y_1),\sigma^{m_2}(y_2),\cdots,\sigma^{m_n}(y_n))$ is almost periodic in $(A_1\times A_2\times\cdots\times A_n$, $\sigma\times\sigma\times\cdots\times\sigma)$ and easily to see that $(\sigma^{m_1}(y_1),\sigma^{m_2}(y_2),\cdots,\sigma^{m_n}(y_n))$ is a almost periodic point of $(\kappa(J),\overline{\sigma})$.

Above all, $(\kappa(J),\overline{\sigma})$ is almost periodically dense.

Proof of (P6). By the proof of (P4), $(\kappa(J),\overline{\sigma})$ is topologically weakly mixing. So it is topologically transitive. By the proof of (P5), $(\kappa(J),\overline{\sigma})$ is almost periodically dense.

Above all, $(\kappa(J),\overline{\sigma})$ is M-system.

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